CBSE Class 10th Maths

Value Based Questions

CHAPTER 7 – COORDINATE GEOMETRY

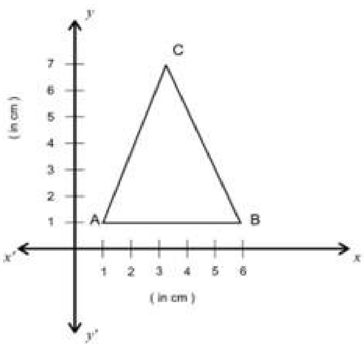
1. **There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. If coordinates of A, B and C are (-2, -3), (2, 3) and (3, 2) respectively then by which bus do you want to travel from A to B (Assume that both buses have same speed.) Which value is depicted in the question?**

   Ans.
   1. By direct route from A to B. Reasoning, Time saving, Economical
   2. (3, 3), Enjoyment, Reasoning.
   3. Rs. 300, Social awareness
   4. Samir, Punctuality, Sincerity.
   5. Rectangular, Economical

2. **In a sports day celebration, Ram and Shyam are standing at positions A and B whose coordinates are (2, -2) and (4, 8) respectively. The teacher asked Geeta to fix the country flag at the mid point of the line joining points A and B. Find the coordinates of the mid point? Which type of value would you infer from the question?**

   Ans. (3, 3), Enjoyment, Reasoning.

3. **To raise social awareness about hazards of smoking, a school decided to start "No Smoking" campaign. 10 students are asked to prepare campaign banners in the shape of triangle as shown in the fig. If cost of 1 cm2 of banner is Rs.2 then find the overall cost incurred on such campaign. Which value is depicted in the question**?

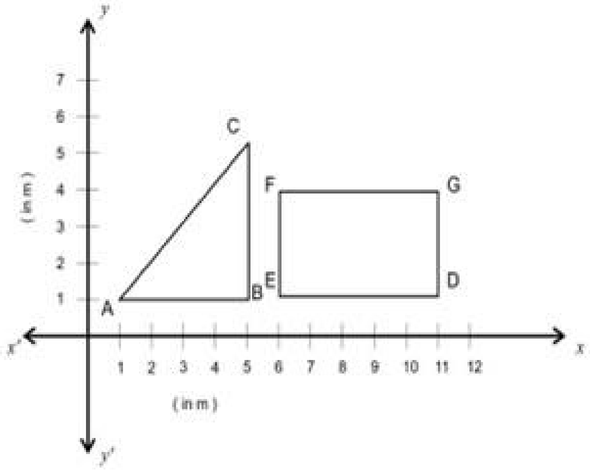


   Ans. Rs. 300, Social awareness

4. **The coordinates of the houses of Sameer and Rahim are (7, 3) and (4, -3) whereas the coordinates of their school is (2, 2). If both leaves their houses at the same time in the morning and also reaches school on time then who travel faster? Which value is depicted in the question?**

   Ans. Samir, Punctuality, Sincerity.

5. **There are two types of fields are available as shown in the fig. which type of field will you purchase if both have same cost? Which value is depicted in the question?**



   Ans. Rectangular, Economical

CBSE Class 10 Mathematics
Important Questions
Chapter 7
Coordinate Geometry

1 Marks Questions

1. The distance between the point $(a,b),(-a,-b)$ is

(a) $2\sqrt{a^2+b^2}$

(b) $2\sqrt{a^2-b^2}$

(c) $\sqrt{a^2+b^2}$

(d) $\sqrt{a+b}$

Ans. $2\sqrt{a^2+b^2}$

2. The area of triangle whose vertices are $(1,-1),(-4,6)$ and $(-3,-5)$ is

(a) 21

(b) 32

(c) 24

(d) 25

Ans. (c) 24

3. The point $(5,-3)$ lies in

(a) 1st quadrant

(b) 2nd quadrant

(c) 3^{rd} quadrant

(d) 4^{th} quadrant

Ans. d) 4^{th} quadrant

4. The distance between the points $(Cos\theta, Sin\theta)$ and $(Sin\theta, -Cos\theta)$ is

(a) $\sqrt{3}$

(b) 2

(c) 1

(d) $\sqrt{2}$

Ans. (d) $\sqrt{2}$

5. If $(1,2)$ $(4,y)$, $(x,6)$ and $(3,5)$ **are the vertices of a parallelogram taken in order. Then** (x,y) **is**

(a) (6, 2)

(b) (6, 3)

(c) (6, 4)

(d) (3, 4)

Ans.(b) (6, 3)

6. The coordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ **in the ratio 2:3 is**

(a) (1, 3)

(b) (2, 3)

(c) (3, 1)

(d) (1, 1)

Ans. (a) (1, 3)

7. The coordinates of a point A, where AB is the diameters of a circle whose centre $(2,-3)$ and B is $(1,4)$ is

(a) (3, -9)

(b) (2, 9)

(c) (3, -10)

(d) (4, 5)

Ans. (c) (3, -10)

8. If the area of a quadrilateral ABCD is zero, then the four points A, B, C, D are

(a) Collinear

(b) Not collinear

(c) Nothing can be said

(d) None of these

Ans. (a) Collinear

9. The valve of K if the points $A(2,3)$, $B(4,K)$ and $C(6,-3)$ are collinear is

(a) (1)

(b) (-1)

(c) (2)

(d) (0)

Ans. (d) (0)

10. The mid-point of the line segment joining $(2a, 4)$ **and** $(-2, 3b)$ **is** $(1, 2a+1)$**. The values of a and b is**

(a) a=2, b=2

(b) a=1, b=3

(c) a=2, b=3

(d) a=1, b=1

Ans. (a) a=2, b=2

11. Coordinate of A and B are $(-3, \alpha)$ **and** $(1, \alpha+4)$. **The mid-point of AB is** $(-1, 1)$**. The value of** α **is**

(a) (-1)

(b) (2)

(c) (3)

(d) (1)

Ans. (a) (-1)

12. The distance between $P(a, 7)$ **and Q** $(1, 3)$ **is 5. The value of a is**

(a) (4, 2)

(b) (-4, -2)

(c) (4, -2)

(d) (4, 1)

Ans. (c) (4, -2)

13. On which axis point $(-4, 0)$ lie

(a) x-axis

(b) y-axis

(c) both

(d) none of these

Ans. (a) x-axis

14. The distance of the point $(-4, -6)$ from the origin is

$(a)\ \sqrt{53}$

$(b)\ 2\sqrt{13}$

$(c)\ 2\sqrt{12}$

$(d)\ \sqrt{13}$

Ans. (b) $2\sqrt{13}$

15. The coordinates of the mid-point of the line segment joining $(-5, 4)$ and $(7, -8)$ is

(a) (1, -2)

(b) (1, 2)

(c) (1, 3)

(d) (-1, -2)

Ans. (a) (1, -2)

16. Two vertices of a ΔABC are $A(1,-1)$ and $B(5,1)$. If the coordinates of its centroid be $\left(\frac{5}{3},1\right)$, then the coordinates of the third vertex C is

(a) (-1, -3)

(b) (1, 3)

(c) (-1, 3)

(d) (1, 2)

Ans. (c) (-1, 3)

17. The abscissa of every point on y-axis is

(a) 0

(b) 1

(c) 2

(d) -1

Ans. (a) 0

18. The ordinate of every point on x-axis is

(a) 1

(b) 2

(c) 0

(d) -1

Ans. (c) 0

19. Find the distance between the following pairs of points:

(i) (2, 3), (4,1)

(ii) (–5, 7), (–1, 3)

(iii) (a, b), (–a, –b)

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get

d = $\sqrt{(4-2)^2+(1-3)^2}$

$=\sqrt{(2)^2+(-2)^2}$

$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$ units

(ii) Applying Distance Formula to find distance between points (–5, 7) and (–1, 3), we get

d = $\sqrt{[-1-(-5)]^2+(3-7)^2}$

$=\sqrt{(4)^2+(-4)^2}$

$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$ units

(iii) Applying Distance Formula to find distance between points (a, b) and (–a, –b), we get

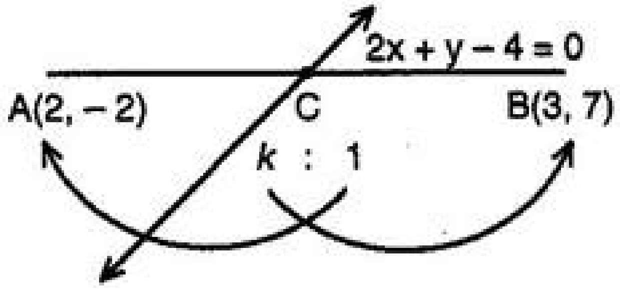
d = $\sqrt{(-a-a)^2+(-b-b)^2}$

$=\sqrt{(-2a)^2+(-2b)^2}=\sqrt{4a^2+4b^2}$

$=\sqrt{4(a^2+b^2)}=2\sqrt{a^2+b^2}$

20. Determine the ratio in which the line 2x+y-4 = 0 divides the line segment joining the points A(2,-2) and B (3,7).

Ans. Let the line $2x+y-4=0$ divides the line segment joining A$(2,-2)$ and B (3, 7) in the ratio $k:1$ at point C. Then, the coordinates of C are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$.



But C lies on $2x+y-4=0$, therefore

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Hence, the required ratio if 2 : 9 internally.

21. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.

Ans. The points A(x, y), B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$$
$$\Rightarrow 2x-y+7y-14=0$$
$$\Rightarrow 2x+6y-14=0$$
$$\Rightarrow x+3y-7=0$$

CBSE Class 10 Mathematics

Important Questions

Chapter 7

Coordinate Geometry

2 Marks Questions

1. Find the distance between the point $A\left(at_1^2, 2at_1\right)$ $B\left(at_2^2, 2at_2\right)$.

Ans. $AB = \sqrt{\left(at_2^2 - at_1^2\right)^2 + \left(2at_2^2 - 2at_1^2\right)^2}$

$= \sqrt{a^2\left(t_2 - t_1\right)^2\left(t_2 + t_1\right)^2 + 4a^2\left(t_2 - t_1\right)^2}$

$= \sqrt{a^2\left(t_2 - t_1\right)^2\left(t_2 + t_1\right)^2 + 4a^2\left(t_2 - t_1\right)^2}$

$= \sqrt{a^2\left(t_2 - t_1\right)^2\left[\left(t_2 + t_1\right)^2 + 4\right]}$

$= a\left(t_2 - t_1\right)\sqrt{\left(t_2 + t_1\right)^2 + 4}$

2. Determine if the points $(1, 5), (2, 3)$ and $(-2, -11)$ **collinear.**

Ans. Let $A = (1,5),\ B = (2,3)\ and\ C = (-2,-11)$

$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{5}$

$BC = \sqrt{(-11-3)^2 + (-2-2)^2} = \sqrt{212}$

$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{265}$

$AB + BC \neq AC$

Hence, A, B and C are not collinear.

3. Prove that the points (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order form a rhombus.

Ans. Let $A\ (3,0),\quad B\ (4,5),\ C\ (-1,4)\ \ and\quad D\ (-2,-1)$

$$AB=\sqrt{(4-3)^2+(5-0)^2}=\sqrt{26}$$

$$BC=\sqrt{(-1-4)^2+(4-5)^2}=\sqrt{26}$$

$$CD=\sqrt{(-2+1)^2+(-1-4)^2}=\sqrt{26}$$

$$DA=\sqrt{(3+2)^2+(0+1)^2}=\sqrt{26}$$

Since AB = BC = CD = DA

Hence, ABCD is a rhombus.

4. Show that (4, 4), (3, 5), (-1, 1) are vertices of a right-angled triangle.

Ans. Let A (4, 4), B (3, 5) and C (-1, 1)

$AB^2 = (3 – 4)^2 + (5 – 4)^2 = 2$

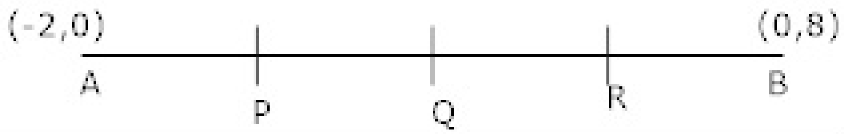
$AC^2 = (-1 – 4)^2 + (5 – 4)^2 = 34$

$BC^2 = (-1 – 3)^2 + (1 – 5)^2=32$

Since $AC^2 = AB^2 + BC^2$

Hence, ABC is a right-angled triangle.

5. Find the coordinates of the points which divide the line segment joining the points (-2, 0) and (0, 8) in four equal parts.



Ans. Q is the mid-point of AB

Coordinate of $Q\left(\frac{-2+0}{2}, \frac{0+8}{2}\right)$ = (-1, 4)

Coordinate of $P = \left(\frac{-3}{2}, 2\right)$

Coordinate of $R = \left(\frac{-1}{2}, 6\right)$

6. Find the area of the rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.

Ans. Let A (3, 0), B (4, 5), C (-1, 4) and D (-2, -1)

$$AC = \sqrt{(-1-3)^2 + (4-0)^2} = 4\sqrt{2}$$

$$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = 6\sqrt{2}$$

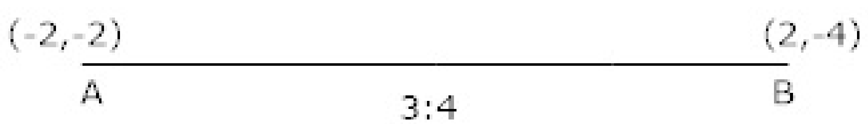
Area of rhombus = $\frac{1}{2} d_1 \times d_2$

$$= \frac{1}{2} AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$$

7. If the coordinates A and B are (-2, -2) and (2, -4) respectively. Find the coordinates of P such that AP = $\frac{3}{7} AB$ and P lies on the line segment AB.

Ans. Coordinate of P are



$$x=\frac{-2\times4+2\times3}{7}=\frac{-8+6}{7}=\frac{-2}{7}$$
$$y=\frac{-2\times4+(-4)\times(3)}{7}=\frac{-8-12}{7}=\frac{-20}{7}$$

8. In what ratio is the line segment joining the points (-2, 3) and (3, 7) divided by the y-axis?

Ans. Let A (-2, -3) and B (3, 7)

P (0, y) and ratio be K:1

Coordinate of P are $\left(\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right)$

$$\frac{3k-2}{k+1}=0$$
$$\Rightarrow k=\frac{2}{3} or\ 2:3$$

9. For what value of P are the points (2, 1) (p, -1) and (-1, 3) collinear?

Ans. For collinear

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=0$$

$$\Rightarrow\frac{1}{2}\left[2(-1-3)+p(3-1)+(-1)(1+1)\right]=0$$

$$\Rightarrow -5+p=0$$
$$\Rightarrow p=5$$

10. Find the third vertex of a Δ, if two of its vertices are at (1, 2) and (3, 5) and the centroid is at the origin.

Ans. Let third vertex of the Δ be (x, y)

$$\frac{x+1+3}{3}=0, \quad \frac{y+2+5}{3}=0$$

$x=-4,\ y=-7$

Hence, third vertex is $(-4,-7)$.

11. In a seating arrangement of desks in a classroom, three students are seated at $A(3,1)$, $B(6,4)$ and $C(8,6)$ respectively. Are they seated in line?

Ans. $AB=\sqrt{(6-3)^2+(4-1)^2}=\sqrt{18}=3\sqrt{2}$

$$BC=\sqrt{(8-6)^2+(6-4)^2}=\sqrt{4+4}=2\sqrt{2}$$

$$AC=\sqrt{(8-3)^2+(6-1)^2}=\sqrt{25+25}=5\sqrt{2}$$

$$AB+BC=AC$$

Hence, they seated in a line.

12. Show that $(1,1), (-1,-1), (\sqrt{3},\sqrt{3})$ are the vertices of an equilateral triangle.

Ans. Let $A(1,1)$, $B(-1,-1)$, $C\left(-\sqrt{3},\sqrt{3}\right)$

$$AB=\sqrt{(-1-1)^2+(-1-1)^2}=\sqrt{8}$$

$$BC=\sqrt{\left(-\sqrt{3}+1\right)2+\left(\sqrt{3}+1\right)2}=\sqrt{8}$$

$$CA=\sqrt{\left(1+\sqrt{3}\right)^2+\left(1-\sqrt{3}\right)^2}=\sqrt{8}$$

Since AB = BC = CA, then ΔABC is equilateral triangle.

13. Find the distance between the points (0, 0) and (36, 15).Also, find the distance between towns A and B if town B is located at 36 km east and15 km north of town A.

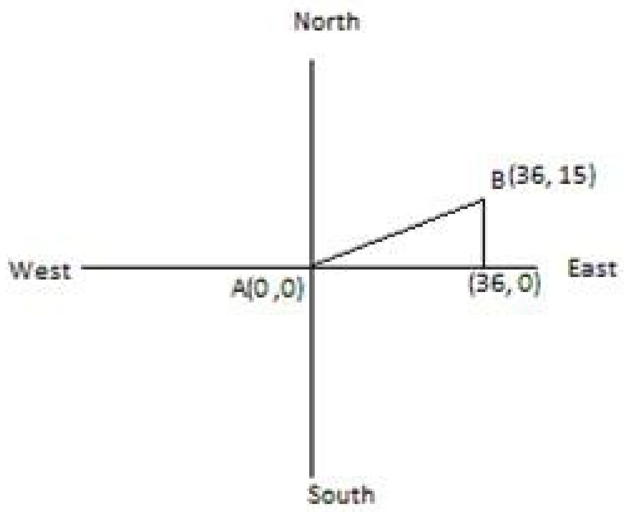
Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

d = $\sqrt{(36-0)^2+(15-0)^2}$

$=\sqrt{(36)^2+(15)^2}$

$=\sqrt{1296+225}=\sqrt{1521}=39$

Town B is located at 36 km east and 15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{[36-0]^2+(15-0)^2}$

$=\sqrt{(36)^2+(15)^2}$

$=\sqrt{1296+225}=\sqrt{1521}=39km$

14. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (–2, –11)

Using Distance Formula to find distance AB, BC and CA.

AB = $\sqrt{[2-1]^2+(3-5)^2}$

$=\sqrt{(1)^2+(-2)^2}$

$=\sqrt{1+4}=\sqrt{5}$

BC = $\sqrt{[-2-2]^2+(-11-3)^2}$

$=\sqrt{(-4)^2+(-14)^2}$

$=\sqrt{16+196}=\sqrt{212}=2\sqrt{53}$

CA = $\sqrt{[-2-1]^2+(-11-5)^2}$

$=\sqrt{(-3)^2+(-16)^2}$

$=\sqrt{9+256}=\sqrt{265}$

Since AB + AC ≠ BC, BC + AC $\neq$ AB and AC $\neq$ BC.

Therefore, the points A, B and C are not collinear.

15. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Ans. Let A = (5, –2), B = (6, 4) and C = (7, –2)

Using Distance Formula to find distances AB, BC and CA.

AB = $\sqrt{[6-5]^2+[4-(-2)]^2}$

$=\sqrt{(1)^2+(6)^2}$

$= \sqrt{1+36} = \sqrt{37}$

BC = $\sqrt{[7-6]^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2}$

$= \sqrt{1+36} = \sqrt{37}$

CA = $\sqrt{[7-5]^2 + [-2-(-2)]^2}$

$= \sqrt{(2)^2 + (0)^2}$

$= \sqrt{4+0} = \sqrt{4} = 2$

Since AB = BC.

Therefore, A, B and C are vertices of an isosceles triangle.

16. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10 units.

Ans. Using Distance formula, we have

$10 = \sqrt{(2-10)^2 + (-3-y)^2}$

$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$

$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$

Squaring both sides, we get

$100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y - 27 = 0$

Solving this Quadratic equation by factorization, we can write

$\Rightarrow y^2+9y-3y-27=0$

$\Rightarrow y(y+9)-3(y+9)=0$

$\Rightarrow (y+9)(y-3)=0$

$\Rightarrow y=3,-9$

17. Find a relation between x and y such that the point (x, y)is equidistant from the point (3, 6) and (–3, 4).

Ans. It is given that (x,y) is equidistant from (3, 6) and (–3, 4).

Using Distance formula, we can write

$$\sqrt{(x-3)^2+(y-6)^2}$$

$$=\sqrt{[x-(-3)]^2+(y-4)^2}$$

$$\Rightarrow \sqrt{x^2+9-6x+y^2+36-12y}$$

$$=\sqrt{x^2+9+6x+y^2+16-8y}$$

Squaring both sides, we get

$\Rightarrow x^2+9-6x+y^2+36-12y$

$=x^2+9+6x+y^2+16-8y$

$\Rightarrow -6x-12y+45$

$=6x-8y+25$

$\Rightarrow 12x+4y=20$

$\Rightarrow 3x+y=5$

18. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the

ratio 2:3.

Ans. Let $x_1=-1$, $x_2=4$, $y_1=7$ and $y_2=-3$, $m_1=2$ and $m_2=3$

Using Section Formula to find coordinates of point which divides join of (–1, 7) and (4, –3) in the ratio 2:3, we get

$$x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$$

$$=\frac{2\times4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$=\frac{2\times(-3)+3\times7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore, the coordinates of point are (1,3) which divides join of (–1, 7) and (4, –3) in the ratio 2:3.

19. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

Ans. Let (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in k:1.

Using Section formula, we get

$$-1=\frac{(-3)\times1+6\times k}{k+1}$$

$\Rightarrow -k-1=(-3+6k)$

$\Rightarrow -7k=-2$

$\Rightarrow k=\frac{2}{7}$

Therefore, the ratio is $\frac{2}{7}$:1 which is equivalent to 2:7.

Therefore, (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in 2:7.

20. Find the ratio in which the line segment joining A(1, –5) and B(–4, 5) is divided by the x–axis. Also find the coordinates of the point of division.

Ans. Let the coordinates of point of division be (x, 0) and suppose it divides line segment joining A(1, –5) and B(–4, 5) in k:1.

According to Section formula, we get

$$x = \frac{1\times1+(-4)\times k}{k+1} = \frac{1-4k}{k+1} \text{ and } 0 = \frac{(-5)\times1+5k}{k+1} \quad \text{... (1)}$$

$$0 = \frac{(-5)\times1+5k}{k+1}$$

⇒ 5=5k

⇒ k=1

Putting value of k in (1), we get

$$x = \frac{1\times1+(-4)\times1}{1+1} = \frac{1-4}{2} = \frac{-3}{2}$$

Therefore, point ($\frac{-3}{2}$, 0) on x–axis divides line segment joining A(1, –5) and B(–4, 5) in 1:1.

21. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Ans. Let A = (1,2), B = (4,y), C= (x, 6) and D= (3, 5)

We know that diagonals of parallelogram bisect each other. It means that coordinates of midpoint of diagonal AC would be same as coordinates of midpoint of diagonal BD. ... (1)

Using Section formula, the coordinates of midpoint of AC are:

$$\frac{1+x}{2}, \frac{2+6}{2} = \frac{1+x}{2}, 4$$

Using Section formula, the coordinates of midpoint of BD are:

$$\frac{4+3}{2}, \frac{5+y}{2} = \frac{7}{2}, \frac{5+y}{2}$$

According to condition (1), we have

$$\frac{1+x}{2} = \frac{7}{2}$$

⇒ (1+x)=7

⇒ x=6

Again, according to condition (1), we also have

$$4 = \frac{5+y}{2}$$

⇒ 8=5+y

⇒ y=3

Therefore, x=6 and y=3

22. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Ans. We want to find coordinates of point A. AB is the diameter and coordinates of center are (2, –3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$$2=\frac{x+1}{2}$$

⇒ 4=x+1

⇒ x=3

Using section formula, we get

$$-3=\frac{4+y}{2}$$

⇒ −6=4+y

⇒ y=−10

Therefore, Coordinates of point A are (3, –10).

23. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order. {Hint: Area of a rhombus = ½ (product of its diagonals)}

Ans. Let A = (3, 0), B = (4, 5), C =(–1, 4) and D = (–2, –1)

Using Distance Formula to find length of diagonal AC, we get

$$AC=\sqrt{[3-(-1)]^2+(0-4)^2}$$

$$=\sqrt{4^2+(-4)^2}$$

$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

Using Distance Formula to find length of diagonal BD, we get

$$BD=\sqrt{[4-(-2)]^2+[5-(-1)]^2}$$

$$=\sqrt{6^2+6^2}$$

$$=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$$

∵ Area of rhombus = ½ (product of its diagonals)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

= 24 sq. units

24. Find the area of the triangle whose vertices are:

(i) (2, 3), (–1, 0), (2, –4)

(ii) (–5, –1), (3, –5), (5, 2)

Ans. (i) (2, 3), (–1, 0), (2, –4)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= ½ [2 {0−(−4)} – 1(−4−3)+2(3−0)]

= ½ [2(0+4) – 1(−7)+2(3)] = ½ (8+7+6)= $\frac{21}{2}$ sq. units

(ii) (–5, –1), (3, –5), (5, 2)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

=½[−5(−5−2)+3 {2−(−1)} +5 {−1−(−5)}]

= ½ [−5(−7)+3(3)+5(4)]

= ½ (35+9+20)

= ½ (64)

=32 sq. units

CBSE Class 10 Mathematics

Important Questions

Chapter 7

Coordinate Geometry

3 Marks Questions

1. If $A(-3,2)$, $B(a,b)$ and $C(-1,4)$ are the vertices of a isosceles triangle, show that $a+b=1$, if AB = BC.

Ans. $AB = BC$ (Given)

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (a+3)^2 + (b-2)^2 = (-1-a)^2 + (4-b)^2$

$\Rightarrow a^2 + 9 + 6a + b^2 + 4 - 4b = 1 + a^2 + 2a + 16 + b^2 - 8b$

$\Rightarrow 4a + 4b = 4$

$\Rightarrow a + b = 1$

2. Find the value of P if the point $A(0,2)$ is equidistant from $(3,p)$ and $(p,3)$.

Ans. Let $B(3,p)$ and $C(p,3)$

AB = AC (Given)

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (0-3)2 + (2-p)^2 = (p-0)^2 + (3-2)^2$

$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 1$

$\Rightarrow -4p = -12$

$\Rightarrow p = 3$

3. Find the centroid of the triangle whose vertices are $(4,-8)(-9,7)$ and $(8,13)$.

Ans. Let (x,y) be the coordinate of centroid

$$x=\frac{x_1+x_2+x_3}{3}$$

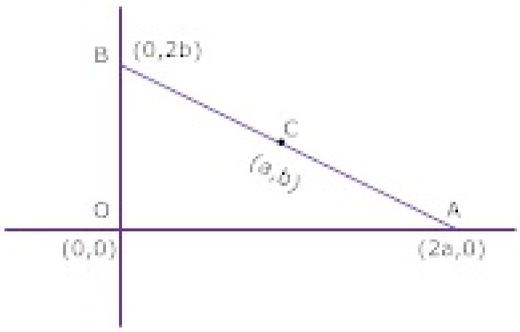
$$=\frac{4-9+8}{3}=\frac{3}{3}=1$$

$$y=\frac{y_1+y_2+y_3}{3}$$

$$=\frac{-8+7+13}{3}=\frac{20-8}{3}=4$$

Coordinate of centroid is $(1,4)$

4. Prove that in a right-angled triangle, the mid-point of the hypotenuse is equidistant from the vertices.



Ans. Let $A(2a,0), B(0,2b)$ and $O(0,0)$ are the vertices of right-angled triangle

Coordinate of $C\left(\frac{2a+0}{2},\frac{0+2b}{2}\right)$

i.e. (a, b)

$$OC = \sqrt{a^2 + b^2}$$

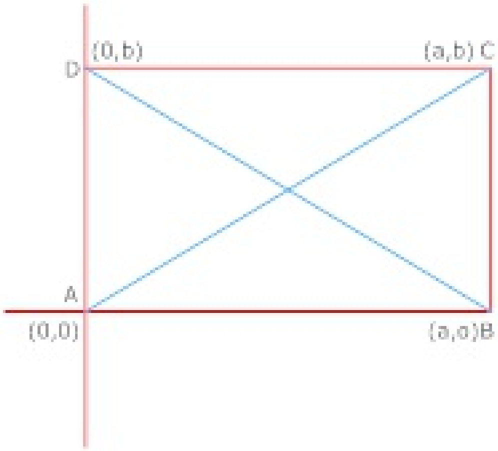
$$AC = \sqrt{a^2 + b^2}$$

$$BC = \sqrt{a^2 + b^2}$$

Hence, C is Equidistant from the vertices.

5. Prove that diagonals of a rectangle bisect each other and are equal.

Ans. Let ABCD be a rectangle take A as origin the vertices of a rectangle are $A(0,0)$, $B(a,0)$, $C(a,b)$, $D(o,b)$



$$AC = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$$

$$BD = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

$$AC = BD$$

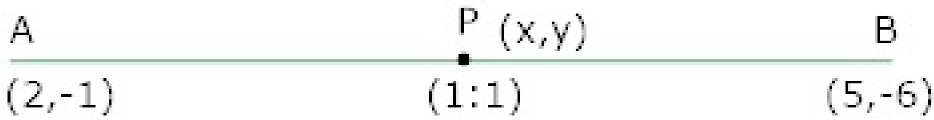
Mid-point of AC = $\left(\frac{0+a}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Mid-point of BD = $\left(\frac{0+a}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Mid-point of AC = Mid-point of BC

Hence proved.

6. The line joining the points $(2,-1)$ and $(5,-6)$ is bisected at P. If P lies on the line $2x+4y+k=0$, find the value of *k*.



Ans. Coordinate of $P=\left(\frac{2+5}{2},\frac{-1-6}{2}\right)=\left(\frac{7}{2},\frac{-7}{2}\right)$

P lies on equation $2x+4y+k=0$

$$\therefore \quad 2\left(\frac{7}{2}\right)+4\left(\frac{-7}{2}\right)+k=0$$

$$\Rightarrow \quad 7-14+k=0$$

$$\Rightarrow \quad k=7$$

7. Show that the points $(a,b+c),(b,c+a)$ and $(c,a+b)$ are collinear.

Ans. For collinear

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

$$=a(c+a-a-b)+b(a+b-b-c)+c(b+c-c-a)$$

$$=a(c-b)+b(a-c)+c(b-a)$$

$$=ac-ab+ba-bc+cb-ca$$

$$=0$$

8. The length of a line segment is 10. If one end point is $(2,-3)$ and the abscissa of the second end point is 10, show that its ordinate is either 3 or -9.

Ans. Let A (2, -3) be the first end point and B (10, y) be the second end point.

$$(10-2)^2+(y+3)^2=(10)^2$$

$$\therefore AB=10$$

$$\Rightarrow \sqrt{(10-2)^2+(y+3)^2}=10$$

$$\Rightarrow 8^2+y^2+9+6y=100$$

$$\Rightarrow 64+y^2+9+6y=100$$

$$\Rightarrow y^2+6y-27=0$$

$$\Rightarrow y^2+9y-3y-27=0$$

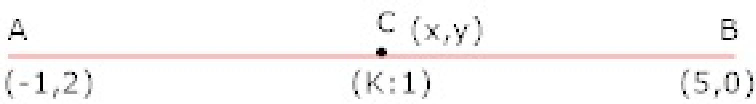
$$\Rightarrow y(y+9)-3(y+9)=0$$

$$\Rightarrow (y+9)(y-3)=0$$

$$\Rightarrow y=-9 \text{ or } y=3$$

9. Using section formula, show that the points $(-1,2)(5,0)$ and $(2,1)$ are collinear.

Ans. If points $A(-1,2)$, $B(5,0)$ and $(2,1)$ are collinear, then one point divides the join of other two in the same ratio. Let $C(2,1)$ divides the join of $A(-1,2)$ and $B(5,0)$ in the ratio K:1



$$\therefore 2=\frac{5K-1}{K+1} \text{ and } 1=\frac{0+2}{K+1}$$

$$2K+2=5K-1 \text{ and } K+1=2$$

$$K=1 \qquad\qquad K=1$$

Hence Proved.

10. Find the relation between x and y such that the point (x,y) is equidistant from the

points $(7,1)$ **and** $(3,5)$.

Ans. Let $P(x,y)$ be equidistant from the points $A(7,1)$ and $B(3,5)$

$AP = BP$ (Given)

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

$\Rightarrow x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

$\Rightarrow x - y = 2$

11. Determine the ratio in which the line $2x + y - 4 = 0$ **divides the line segment joining the points** $A(2,2)$ **and** $B(3,7)$.

Ans. Let the ratio be K: 1

Coordinate of P are $\left(\frac{3K+2}{K+1}, \frac{7K-2}{K+1}\right)$

P lies on the line $2x + y - 4 = 0$

$\Rightarrow 2\left(\frac{3K+2}{K+1}\right) + \frac{7K-2}{K+1} - \frac{4}{1} = 0$

$\Rightarrow 6K + 4 + 7K - 2 - 4K - 4 = 0$

$\Rightarrow 9K - 2 = 0$

$\Rightarrow K = \frac{2}{9} \text{ or } 2:9$

12. Show that the points $A(5,6)$, $B(1,5)$, $C(2,1)$ **and** $D(6,2)$ **are the vertices of a square.**

Ans. $AB = \sqrt{(1-5)^2 + (5-6)^2} = \sqrt{17}$

$$BC=\sqrt{(2-1)^2+(1-5)^2}=\sqrt{17}$$
$$CD=\sqrt{(6-2)^2+(1-2)^2}=\sqrt{17}$$
$$DA=\sqrt{(5-6)^2+(6-2)^2}=\sqrt{17}$$

Diagonal $AC=\sqrt{(2-5)^2+(1-6)^2}=\sqrt{34}$

Diagonal $BD=\sqrt{(6-1)^2+(2-5)^2}=\sqrt{34}$

Hence proved.

13. If the point $P(x,y)$ is equidistant from the points $A(5,1)$ and $B(1,5)$, prove that $x=y$.

Ans. $PA=PB$ (Given)

$$\therefore PA^2=PB^2$$
$$\Rightarrow (5-x)^2+(1-y)^2=(1-x)^2+(5-y)^2$$
$$\Rightarrow 25+x^2-10x+1+y^2-2y=1+x^2-2x+25+y^2-10y$$
$$\Rightarrow -8x=-10y+2y$$
$$\Rightarrow -8x=-8y$$
$$\Rightarrow x=y$$

14. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).

Ans. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9).

Using Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2+[0-(-5)]^2}=\sqrt{[x-(-2)]^2+[(0-9)]^2}$$

$$\Rightarrow \sqrt{x^2+4-4x+25}=\sqrt{x^2+4+4x+81}$$

Squaring both sides, we get

⇒ $x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

⇒ $-4x + 29 = 4x + 85$

⇒ $8x = -56$

⇒ $x = -7$

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

15. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Ans. We want to find coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

We are given AC = CD = DB

We want to find coordinates of point C and D.

Let coordinates of point C be (x_1, y_1) and let coordinates of point D be (x_2, y_2).

Clearly, point C divides line segment AB in 1:2 and point D divides line segment AB in 2:1.

Using Section Formula to find coordinates of point C which divides join of (4, –1) and (–2, –3) in the ratio 1:2, we get

$$x_1 = \frac{1\times(-2)+3\times4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1\times(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

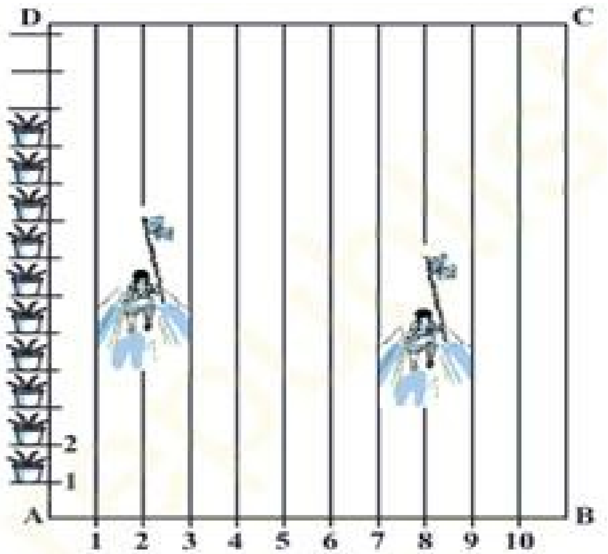
Using Section Formula to find coordinates of point D which divides join of (4, –1) and (–2, –3) in the ratio 2:1, we get

$$x_2 = \frac{2\times(-2)+1\times4}{1+2} = \frac{-4+4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2\times(-3)+1\times(-1)}{1+2} = \frac{-6-1}{3} = \frac{-7}{3}$$

Therefore, coordinates of point C are (2, $\frac{-5}{3}$) and coordinates of point D are (0, $\frac{-7}{3}$).

16. To conduct sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag. Preet runs 15th of the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Ans. Niharika runs 14^{th} of the distance AD on the 2^{nd} line and posts a green flag.

There are 100 flower pots. It means, she stops at 25th flower pot.

Therefore, the coordinates of point where she stops are (2 m, 25 m).

Preet runs 15th of the distance AD on the eighth line and posts a red flag. There are 100 flower pots. It means, she stops at 20th flower pot.

Therefore, the coordinates of point where she stops are (8, 20).

Using Distance Formula to find distance between points (2 m, 25 m) and (8 m, 20 m), we get

$$d = \sqrt{(2-8)^2 + (25-20)^2} = \sqrt{(-6)^2 + 5^2} = \sqrt{36+25} = \sqrt{61}m$$

Rashmi posts a blue flag exactly halfway the line segment joining the two flags.

Using section formula to find the coordinates of this point, we get

$$x = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25+20}{2} = \frac{45}{2}$$

Therefore, coordinates of point, where Rashmi posts her flag are (5, $\frac{45}{2}$).

It means she posts her flag in 5th line after covering $\frac{45}{2}$ = 22.5 m of distance.

17. If A and B are (–2, –2) and (2, –4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

Ans. A = (–2, –2) and B=(2, –4)

It is given that AP= $\frac{3}{7}$ AB

PB=AB – AP=AB– $\frac{3}{7}$ AB= $\frac{4}{7}$ AB

So, we have AP:PB = 3:4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

$$x = \frac{(-2)\times 4 + 2\times 3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2)\times 4 + (-4)\times 3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

18. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, –2), (5, 1), (3, k)

(ii) (8, 1), (k, –4), (2, –5)

Ans. (i) (7, –2), (5, 1), (3, k)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

⇒ ½ [7(1−k)+5 {k−(−2)} +3(−2−1)] = ½ (7−7k+5k+10−9)=0

⇒ ½ (7−7k+5k+1)=0

⇒ ½ (8−2k)=0

⇒ 8−2k=0s

⇒ 2k=8

⇒ k=4

(ii) (8, 1), (k, –4), (2, –5)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

⇒ ½ [8 {−4−(−5)} +k(−5−1)+2 {1−(−4)}] = ½ (8−6k+10)=0

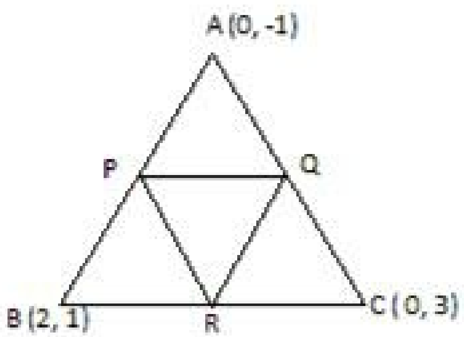
⇒ ½ (18−6k)=0

⇒ 18−6k=0

⇒ 18=6k

⇒ k=3

19. Find the area of the triangle formed by joining the mid–points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle



Ans. Let A = (0, –1) = (x_1, y_1), B = (2, 1) = (x_2, y_2) and

C = (0, 3) = (x_3, y_3)

Area of △ABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

⇒ Area of △ABC

$= \frac{1}{2}[0(1-3)+2\{3-(-1)\}+0(-1-1)] = \frac{1}{2} \times 8$

=4 sq. units

P, Q and R are the mid–points of sides AB, AC and BC respectively.

Applying Section Formula to find the vertices of P, Q and R, we get

$$P = \frac{0+2}{2}, \frac{1-1}{2} = (1,0)$$

$$Q = \frac{0+0}{2}, \frac{-1+3}{2} = (0,1)$$

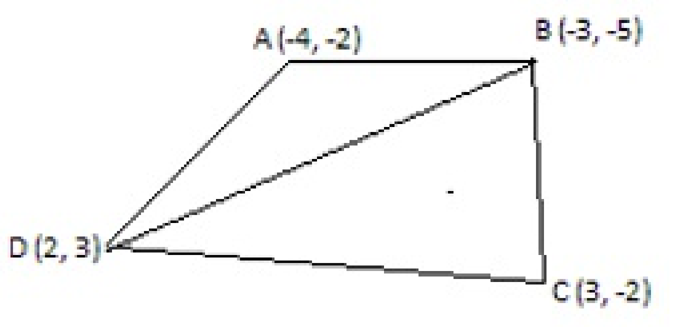
$$R = \frac{2+0}{2}, \frac{1+3}{2} = (1,2)$$

Applying same formula, Area of $\triangle$PQR= $\frac{1}{2}[1(1-2)+0(2-0)+1(0-1)] = \frac{1}{2}\left|-2\right|$

=1 sq. units (numerically)

Now, $\frac{Area\ of\ \Delta PQR}{Area\ of\ \Delta ABC} = \frac{1}{4} = 1:4$

20. Find the area of the quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3).



Ans. Area of Quadrilateral ABCD

= Area of Triangle ABD + Area of Triangle BCD … (1)

Using formula to find area of triangle:

Area of $\triangle$ABD

= $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

= ½ [−4(−5−3) – 3 {3−(−2)} +2 {−2−(−5)}]

= ½ (32 – 15+6)= ½ (23)=11.5 sq units … (2)

Again using formula to find area of triangle:

Area of △BCD = $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

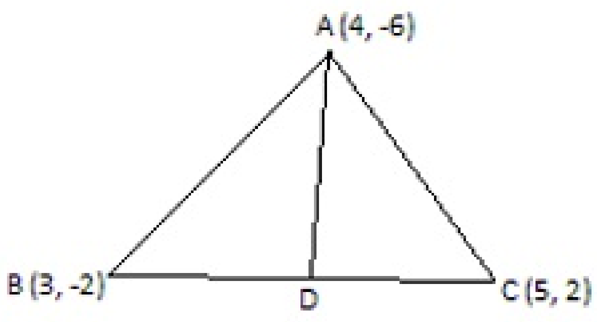
= ½ [−3(−2−3)+3 {3−(−5)} +2 {−5−(−2)}]

= ½ (15+24−6)= ½ (33)=16.5 sq units … (3)

Putting (2) and (3) in (1), we get

Area of Quadrilateral ABCD = 11.5 + 16.5 = 28 sq units.

21. We know that median of a triangle divides it into two triangles of equal areas. Verify this result for △ABC whose vertices are A (4, –6), B (3, –2) and C (5, 2).



Ans. We have △ABC whose vertices are given.

We need to show thatar(△ABD) = ar(△ACD).

Let coordinates of point D are (x, y)

Using section formula to find coordinates of D, we get

$$x = \frac{3+5}{2} = \frac{8}{2} = 4$$

$$y = \frac{-2+2}{2} = \frac{0}{2} = 0$$

Therefore, coordinates of point D are (4, 0)

Using formula to find area of triangle:

Area of △ABD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= ½ [4(−2−0)+3 {0−(−6)} +4 {−6−(−2)}]

= ½ (−8+18−16)= ½ (−6)=−3 sq units

Area cannot be in negative.

Therefore, we just consider its numerical value.

Therefore, area of △ABD = 3 sq units ... (1)

Again using formula to find area of triangle:

Area of △ACD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

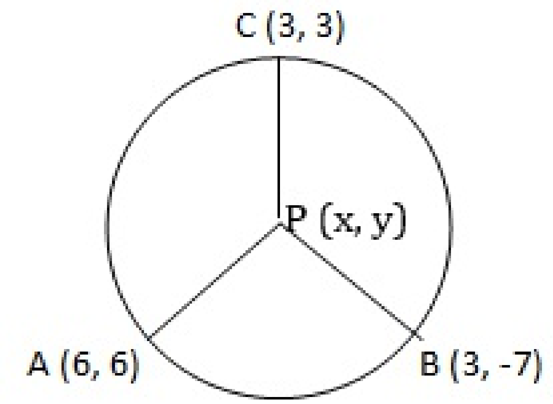
= ½ [4(2−0)+5 {0−(−6)} +4 {−6−2)}]

= ½ (8+30−32)= ½ (6)=3 sq units ... (2)

From (1) and (2), we getar(△ABD) = ar(△ACD)

Hence Proved.

22. Find the centre of a circle passing through the points $(6, -6), (3, -7)$ **and** $(3, 3)$.



Ans. Let P(x, y), be the centre of the circle passing through the points A$(6, -6)$, B$(3, -7)$ and C (3, 3). Then AP = BP = CP.

Taking AP = BP

$\Rightarrow$ $AP^2 = BP^2$

$\Rightarrow (x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow -12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow -6x - 2y + 14 = 0$

$\Rightarrow 3x + y - 7 = 0$ ....(i)

Again, taking BP = CP

$\Rightarrow$ $BP^2 = CP^2$

$\Rightarrow (x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$

$\Rightarrow -6x + 6x + 14y + 6y + 58 - 18 = 0$

$\Rightarrow 20y + 40 = 0$

$\Rightarrow y = -2$

Putting the value of y in eq. (i),

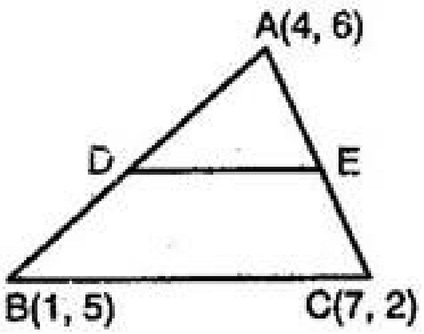
$3x + y - 7 = 0$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

Hence, the centre of the circle is $(3, -2)$.

23. The vertices of a $\triangle$ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the $\triangle$ADE and compare it with the area of $\triangle$ABC.



Ans. Since, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$\therefore$ DE $\parallel$ BC [By Thales theorem]

$\therefore$ $\triangle$ADE $\sim$ $\triangle$ABC

$$\therefore \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$= \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} \quad \text{..........(i)}$$

Now, Area (ΔABC) = $\frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$

= $\frac{1}{2}[12-4+7]=\frac{15}{2}$ sq. units ....(ii)

From eq. (i) and (ii),

Area (ΔADE) = $\frac{1}{16}\times$ Area (ΔABC) = $\frac{1}{16}\times\frac{15}{2}=\frac{15}{32}$ sq. units

$\therefore$ Area (ΔADE) : Area (ΔABC) = 1 : 16

CBSE Class 10 Mathematics

Important Questions

Chapter 7

Coordinate Geometry

4 Marks Questions

1. If the points (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that $bx = ay$.

Ans. Let $P(x, y)$, $A(a+b, b-a)$ and $B(a-b, a+b)$

$PA = PB \quad (\text{Given})$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow a^2 + b^2 + x^2 + 2ab - 2ax - 2ax + b^2 + a^2 + y^2 - 2ab + 2ay - 2by$

$\Rightarrow a^2 + b^2 + x^2 - 2ab + 2bx - 2ax + a^2 + b^2 + y^2 + 2ab - 2by - 2ay$

$\Rightarrow 4ab - 4bx - 4ab = -2ay - 2ay$

$\Rightarrow -4bx = -4ay$

$\Rightarrow bx = ay$

2. $(-2, 2)$, $(x, 8)$ and $(6, y)$ are three concyclic points whose centre is $(2, 5)$. Find the possible value of x and y.

Ans. $OA = OB = OC =$ Radius of circle

$\Rightarrow OA^2 = OB^2 = OC^2$

$OB^2 = OA^2$

$\Rightarrow (x-2)^2 + (8-5)^2 = (2+2)^2 + (5-2)^2$

$\Rightarrow x^2 + 4 - 4x + 9 = 16 + 9$

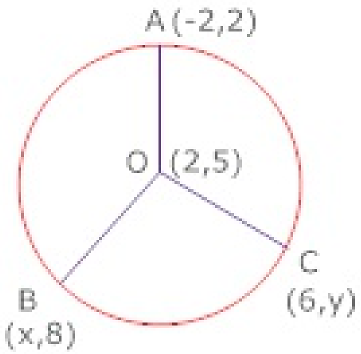
$\Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x-6) + 2(x-6) = 0$

$\Rightarrow (x-6)(x+2) = 0$

$\Rightarrow x = 6 \text{ or } x = -2$



$OC^2 = OA^2$

$\Rightarrow (6-2)^2 + (y-5)^2 = (2+2)^2 + (5-2)^2$

$\Rightarrow (4)^2 + y^2 + 25 - 10y = 16 + 9$

$\Rightarrow y^2 - 10y + 16 = 0$

$\Rightarrow y^2 - 8y - 2y + 16 = 0$

$\Rightarrow y(y-8) - 2(y-8) = 0$

$\Rightarrow (y-8)y - 2 = 0$

$\Rightarrow y = 8 \text{ or } y = 2$

3. Find the vertices of the triangle, the mid-points of whose sides are $(3,1)$, $(5,6)$ **and** $(-3,2)$.

Ans. Let vertices of ΔABC be $A(x_1 y_1), B(x_2 y_2)$ and $C(x_3, y_3)$

By mid-points formula

$$\frac{x_2 + x_3}{2} = 3 \Rightarrow x_2 + x_3 = 6 \ldots\ldots.(i)$$

$$\frac{y_2 + y_3}{2} = 1 \Rightarrow y_2 + y_3 = 2 \ldots\ldots\ldots(ii)$$

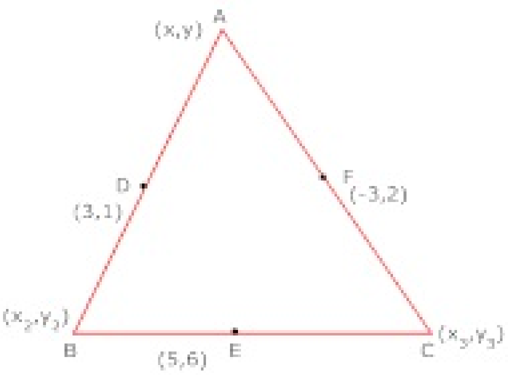
$$\frac{x_3 + x_1}{2} = 5 \Rightarrow x_3 + x_1 = 10 \ldots\ldots\ldots(iii)$$

$$\frac{y_3 + y_1}{2} = 6 \Rightarrow y_1 + y_3 = 12 \ldots\ldots\ldots(iv)$$

$$\frac{x_1 + x_2}{2} = -3 \Rightarrow x_1 + x_2 = -6 \ldots\ldots\ldots(v)$$

$$\frac{y_1 + y_2}{2} = 2 \Rightarrow y_1 + y_2 = 4 \ldots\ldots\ldots(vi)$$

Adding (i), (iii) and (v)



$$2(x_1 + x_2 + x_3) = 10$$
$$\Rightarrow x_1 + x_2 + x_3 = 5 \ldots\ldots\ldots(vii)$$

Adding (ii), (iv)and (vi)

$$2(y_1 + y_2 + y_3) = 18$$
$$y_1 + y_2 + y_3 = 9 ....(viii)$$

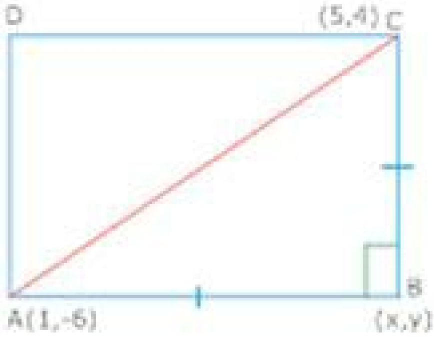
Subtracting (i), (iii) and (v) from (vii)

We get, $x_1 = -1, x_2 = -5, x_3 = 11$

Subtracting (ii), (iv) and (vi) from eq. (viii)

We get, $y_1 = 7, y_2 = -3, y_3 = 5$

4. The two opposite vertices of a square are $(1, -6)$ and $(5, 4)$. Find the coordinates of the other two vertices.



Ans. $AB = BC$

$$\Rightarrow AB^2 = BC^2$$
$$\Rightarrow (x-1)^2 + (y+6)^2 = (x-5)^2 + (y-4)^2$$
$$\Rightarrow x^2 + 1 - 2x + y^2 + 36 + 12y = x^2 + 25 - 10x + y^2 + 16 - 8y$$

$$\Rightarrow 8x + 20y - 4 = 0$$
$$\Rightarrow 2x + 5y = 1$$
$$\Rightarrow y = \frac{1-2x}{5}$$

In right ΔABC,

$$AC^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (x-1)^2 + (y+6)^2 + (x-5)^2 + (y-4)^2 = (5-1)^2 + (4+6)^2$$

$$\Rightarrow 2(x^2 + y^2 - 6x + 2y) = 38$$

$$\Rightarrow x^2 + y^2 - 6x + 2y = 19 ....(i)$$

Put the value of y in eq. (i)

$$x^2 + \left(\frac{1-2x}{5}\right)^2 - 6x + 2\left(\frac{1-2x}{5}\right) = 19$$

$$\Rightarrow 29x^2 - 174x - 464 = 0$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow x^2 - 8x + 2x - 16 = 0$$

$$\Rightarrow x(x-8) + 2(x-8) = 0$$

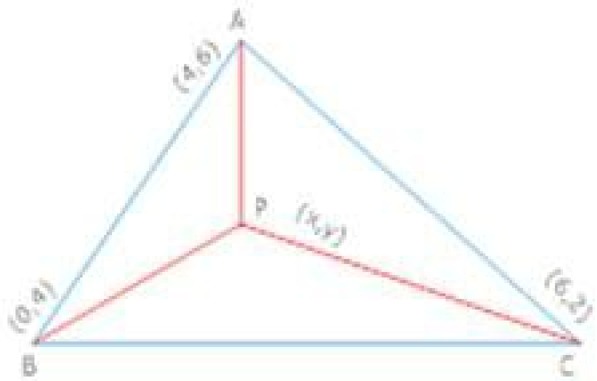
$$\Rightarrow (x-8)(x+2) = 0$$

$$\Rightarrow x = 8 \; or \; x = -2$$

Now $x = -2, \Rightarrow y = 1$

And $x = 8 \Rightarrow y = -3$

5. Find the coordinates of the circumcentre of a triangle whose vertices are A(4,6), B(0,4) and C(6,2). Also find its circum-radius.



Ans. Let P be the circum-centre of ΔABC, then PA = PB = PC

$$\Rightarrow PA^2 = PB^2 = PC^2$$

$$PA^2 = PB^2$$

$$\Rightarrow (x-4)^2 + (y-6)^2 = (x-0)^2 + (y-4)^2$$

$$\Rightarrow 8x + 4y = 36$$

$$\Rightarrow 2x + y = 9 ....(i)$$

$$PB^2 = PC^2$$

$$\Rightarrow (x-0)^2 + (y-4)^2 = (x-6)^2 + (y-2)^2$$

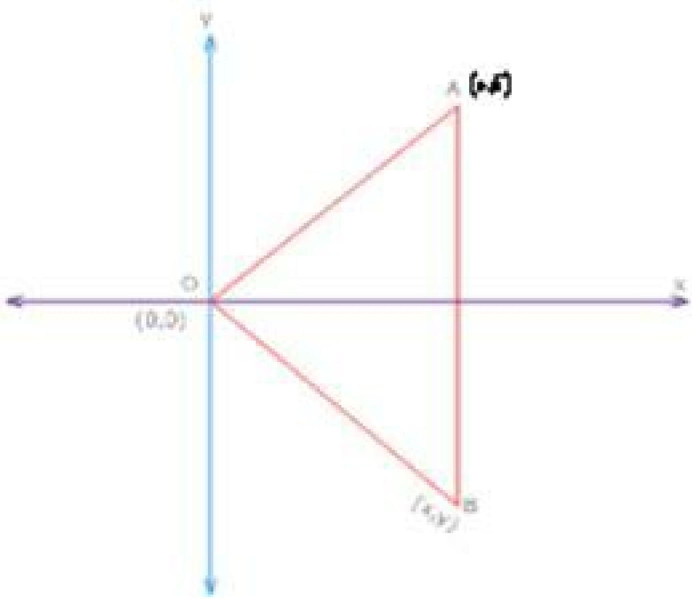
$$\Rightarrow 12x - 4y = 24$$

$$\Rightarrow 3x - y = 6 ....(ii)$$

On solving equations (i) and (ii), $x = 3, y = 3$

Circum-radius (PA) = $\sqrt{(4-3)^2 + (6-3)^2} = \sqrt{10}$

6. If two vertices of an equilateral triangle are $(0,0)\left(3,\sqrt{3}\right)$, Find the third vertex.



Ans. $OA = OB = AB$

$\Rightarrow OA^2 = OB^2 = AB^2$

$OA^2 = OB^2$

$\Rightarrow x^2 + y^2 = 12 ....(i)$

$OB^2 = AB^2$

$\Rightarrow 3x + \sqrt{3}y = 6 ....(ii)$

$$\Rightarrow y = \frac{6-3x}{\sqrt{3}}$$

Put the value of y in eq. (i),

$$x^2 + \left(\frac{6-3x}{\sqrt{3}}\right)^2 = 12$$

$\Rightarrow x = 0 \quad or \quad x = 3$

$When\ x = 0, y = 2\sqrt{3}$

$\left(0, 2\sqrt{3}\right)$

$When\ x = 3, y = -\sqrt{3}$

$\left(3, -\sqrt{3}\right)$

7. If P and Q are two points whose coordinates are $\left(at^2, 2at\right)$ and $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ respectively and S is the point $\left(a, 0\right)$, show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of t.

Ans. $SP = \sqrt{\left(at^2 - a\right)^2 + \left(2at - 0\right)^2}$

$$= a\sqrt{\left(t^2-1\right)^2+4t^2}$$

$$= a\sqrt{t^4+1-2t^2+4t^2}$$

$$= a\sqrt{\left(t^2+1\right)^2}$$

$$= a\left(t^2+1\right)$$

$$SQ = \sqrt{\left(\frac{a}{t^2}-a\right)^2+\left(\frac{2at}{t}-0\right)^2}$$

$$= \frac{a}{t^2}\sqrt{\left(1+t^2\right)^2}$$

$$= \frac{a}{t^2}\left(1+t^2\right)$$

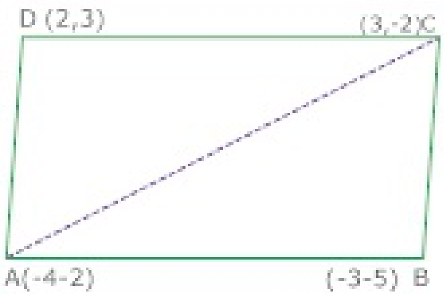
$$\frac{1}{SP}+\frac{1}{SQ} = \frac{1}{a\left(t^2+1\right)}+\frac{t^2}{a\left(1+t^2\right)}$$

$$= \frac{\left(1+t^2\right)}{a\left(t^2+1\right)}$$

$$= \frac{1}{a}$$

Hence proved.

8. Find the area of the quadrilateral whose vertices taken in order are (-4,-2), (-3,5), (3,-2) and (2,3).



Ans. $ar(\Delta ABC) = \frac{1}{2}[(20+6-6)-(6-15+8)]$

= 10.5 sq. units

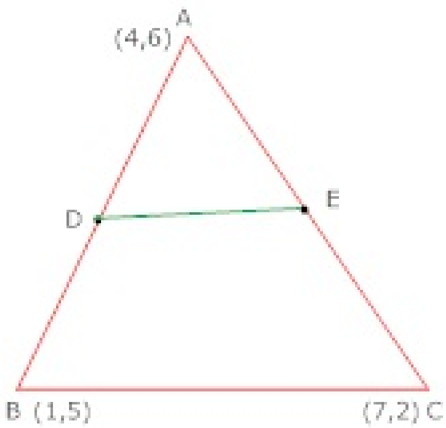
$$\left[ar \text{ of } \Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$ar(\Delta ACD) = \frac{1}{2}[(8+9-4)-(-6-4-12)]$$

$$= \frac{1}{2}[13+22] = 17.5 \text{ sq.units}$$

area of quadrilateral = 10.5 + 17.5 = 28 sq. units.

9. The vertices of ΔABC are $A(4,6), B(1,5)$ and $C(7,2)$. A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC.



Ans. $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = \frac{4}{1}$$

$$\frac{AD+DB}{AD} = \frac{AE+EC}{AE} = 4$$

$$1+\frac{DB}{AD} = 1+\frac{EC}{AE} = 4$$

$$\frac{DB}{AD} = \frac{EC}{AE} = 3$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

$$AD:DB = AE:EC = 1:3$$

Now coordinate of D and E are

$\left(\frac{13}{4}, \frac{23}{4}\right)$ and $\left(\frac{19}{4}, 5\right)$

$$ar(\Delta ADE) = \frac{15}{32}$$

$$ar(\Delta ABC) = \frac{15}{2}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{1}{16}$$

$$= 1:16$$

10. Prove that the points $(a, a), (-a, -a)$ and $\left(-\sqrt{3}a, \sqrt{3}a\right)$ are the vertices of an equilateraltriangle. Calculate the area of this triangle.

Ans. Let $A(a, a), B(-a, -a)$ $C\left(-\sqrt{3}a, \sqrt{3}a\right)$

$$AB = \sqrt{(-a-a)^2 + (-a-a)^2} = \sqrt{8a^2} = 2\sqrt{2}a$$

$$AB = \sqrt{\left(-\sqrt{3a}+a\right)^2 + \left(\sqrt{3}a+a\right)^2} = 2\sqrt{2}a$$

$$AC = \sqrt{\left(-\sqrt{3}a-a\right)^2 + \left(\sqrt{3}a-a\right)^2} = 2\sqrt{2}a$$

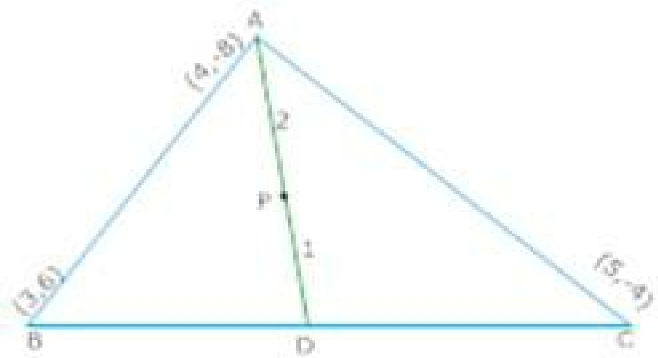
$$\therefore AB = BC = AC = 2\sqrt{2}a$$

$$ar\Delta ABC = \frac{\sqrt{3}}{4}\times(side)^2$$

$$= \frac{\sqrt{3}}{4}\times\left(2\sqrt{2}a\right)$$

$$= 2\sqrt{3}a^2$$

11. $A(4,-8)$, $B(3,6)$ and $C(5,-4)$ **are the vertices of a** ΔABC, D **is the mid-point of BC and P is a point on AD joined such that** $\frac{AD}{PD} = 2$, **find the coordinates of P.**



Ans. Let $A(4,-8)$, $B(3,6)$ and $C(5,-4)$ are the vertices of ΔABC, D is the mid- point of BC

$$\frac{AP}{PD} = \frac{2}{1}$$

$$\Rightarrow AP : PD = 2:1$$

Coordinate of D $\left(\frac{3+5}{2}, \frac{6-4}{2}\right)$

i.e., $(4,1)$

Coordinate of P are $\left(\frac{2\times4+1\times4}{2+1}, \frac{2\times1+1\times(-8)}{2+1}\right)$

$i.e., \left(\frac{8+4}{3}, \frac{2-8}{3}\right)$

$i.e., (4,-2)$

12. The coordinates of the vertices of ΔABC are $A(4,1), B(-3,2)$ and $C(O,K)$. Given that the area of ΔABC is 12, find the value of K.

Ans. $A(4,1), B(-3,2)$ and $C(0,k)$

$$ar\Delta ABC = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[4(2-k)+(-3)(k-1)+0(1-2)\right]$$

$$=\frac{1}{2}[8-4k-3k+3]=\frac{1}{2}[11-7k]$$

But $area\ of\ \Delta = 12$

$$\Rightarrow \frac{1}{2}|11-7k|=12$$

$$\Rightarrow \frac{1}{2}(11-7k)=\pm12$$

$$\Rightarrow 11-7k=24$$

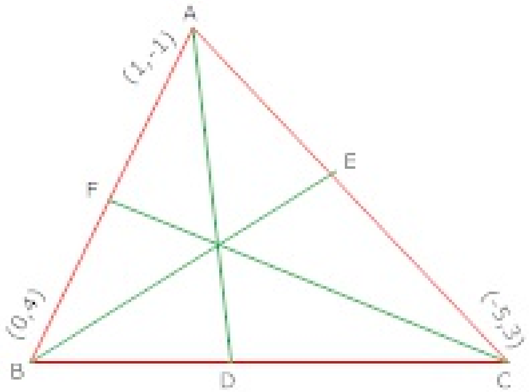
$\Rightarrow k = \frac{-13}{7}$

$If\ 11 - 7k = -24$

$\Rightarrow k = 5$

Value of $k\left(\frac{-13}{7}, 5\right)$

13. Find the lengths of the medians of the triangle whose vertices are $(1, -1), (0, 4)$ and $(-5, 3)$.



Ans. Coordinates of points D, E and F are

$$\left(\frac{0-5}{2}, \frac{4+3}{2}\right), \left(\frac{-5+1}{2}, \frac{3-1}{2}\right) \text{and} \left(\frac{1+0}{2}, \frac{-1+4}{2}\right)$$

$$i.e \left(\frac{-5}{2}, \frac{7}{2}\right), (-2, 1) \text{ and} \left(\frac{1}{2}, \frac{3}{2}\right)$$

Length of the median AD

$$= \sqrt{\left(\frac{-5}{2} - 1\right)^2 + \left(\frac{7}{2} + 1\right)^2} = \frac{\sqrt{130}}{2}$$

Length of the median BE

$$= \sqrt{(-2-0)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$$

And length of the median CF

$$CF=\sqrt{\left(\frac{1}{2}+5\right)^2+\left(\frac{3}{2}-3\right)^2}=\frac{\sqrt{130}}{2}$$

14. The area of a triangle is 5. Two of its vertices are $(2,1)$ and $(3,-2)$. The third vertex lies on $y=x+3$. Find the third vertex.

Ans. Let the third vertex be $A(x,y)$. Other two vertices of the Δ are $B(2,1)$ and $C(3,-2)$

ar of $\Delta ABC=5$

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=\pm 5$$

$$\Rightarrow \frac{1}{2}\left[x(1+2)+2(-2-y)+3(y-1)\right]=\pm 5$$

$$\Rightarrow 3x+y-7=\pm 10$$
$$\Rightarrow 3x+y=17 \ or\ 3x+y=-3$$

(x,y) lies on eq. $y=x+3$

On solving eq. $3x+y=17$and $y=x+3$

We get $x=\frac{7}{2}, y=\frac{13}{2}$

Similarly, on solving eq. $3x+y=-3 \ and \ y=x+3$

We get $\left(\frac{-3}{2},\frac{3}{2}\right)$

15. Prove that the point $(a,o),(a,b)$ and $(1,1)$ are collinear, if $\frac{1}{a}+\frac{1}{b}=1$

Ans. Since $(a, 0), (0, b)$ and $(1, 1)$ are collinear

Area = 0

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$$

$$\Rightarrow \frac{1}{2}\left[a(b-1) + 0(1-0) + 1(0-b)\right] = 0$$

$$\Rightarrow ab - a - b = 0$$

$$\Rightarrow ab = a + b$$

Dividing by ab,

$$\frac{ab}{ab} = \frac{a}{ab} + \frac{b}{ab}$$

$$\frac{1}{a} + \frac{1}{b} = 1$$

16. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. UsingDistance Formula, we get

PQ = RQ

$\Rightarrow$ $PQ^2 = RQ^2$

$$\Rightarrow \sqrt{(0-5)^2 + \left[1-(-3)^2\right]} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\Rightarrow \sqrt{(-5)^2 + \left[4^2\right]} = \sqrt{(x)^2 + (-5)^2}$$

$$\Rightarrow \sqrt{25+16} = \sqrt{x^2 + 25}$$

Squaring both sides, we get

$\Rightarrow 25+16=x^2+25$

$\Rightarrow x^2=16$

$\Rightarrow x=4,-4$

Thus, Q is (4, 6) or (–4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4

QR= $\sqrt{(4-0)^2+\left[6-1^2\right]}$

$=\sqrt{16+25}=\sqrt{41}$

Using value of x = –4

QR= $\sqrt{(-4-0)^2+\left[6-1^2\right]}$

$=\sqrt{16+25}=\sqrt{41}$

Therefore, QR= $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4

PR= $\sqrt{(4-5)^2+\left[6-(-3)^2\right]}$

$=\sqrt{1+81}=\sqrt{82}$

Using value of x =–4

PR= $\sqrt{(-4-5)^2+\left[6-(-3)^2\right]}$

$=\sqrt{81+81}=\sqrt{162}=9\sqrt{2}$

Therefore, x = 4, –4

QR= $\sqrt{41}$, PR= $\sqrt{82}, 9\sqrt{2}$

17. Find the coordinates of the points which divides the line segment joining A(–2, 2) and B(2, 8) into four equal parts.

Ans. A = (–2, 2) and B = (2, 8)

Let P, Q and R are the points which divide line segment AB into 4 equal parts.

Let coordinates of point P =(x_1, y_1), Q =(x_2, y_2) and R =(x_3, y_3)

We know AP = PQ = QR = RS.

It means, point P divides line segment AB in 1:3.

Using Section formula to find coordinates of point P, we get

$$x_1 = \frac{(-2)\times 3 + 2\times 1}{1+3} = \frac{-6+2}{4} = \frac{-4}{4} = -1$$

$$y_1 = \frac{2\times 3 + 8\times 1}{1+3} = \frac{6+8}{4} = \frac{14}{4} = \frac{7}{2}$$

Since, AP = PQ = QR = RS.

It means, point Q is the mid-point of AB.

Using Section formula to find coordinates of point Q, we get

$$x_2 = \frac{(-2)\times 1 + 2\times 1}{1+1} = \frac{-2+2}{2} = \frac{0}{2} = 0$$

$$y_2 = \frac{2\times 1 + 8\times 1}{1+1} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Because, AP = PQ = QR = RS.

It means, point R divides line segment AB in 3:1

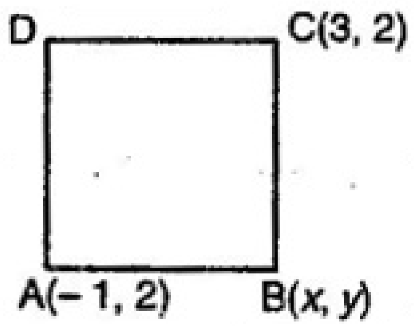
Using Section formula to find coordinates of point P, we get

$$x_3=\frac{(-2)\times1+2\times3}{1+3}=\frac{-2+6}{4}=\frac{4}{4}=1$$

$$y_3=\frac{2\times1+8\times3}{1+3}=\frac{2+24}{4}=\frac{26}{4}=\frac{13}{2}$$

Therefore, P=(–1, $\frac{7}{2}$), Q= (0, 5)and R =(1, $\frac{13}{2}$)

18. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.



Ans. Let ABCD be a square and B(x,y) be the unknown vertex.

AB = BC

$\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow (x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow 2x+1=-6x+9$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(i)

In ΔABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow (x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2=(3+1)^2+(2-2)^2$

$\Rightarrow 2x^2+2y^2+2x-4y-6x-4y+1+4+9+4=16$

$\Rightarrow 2x^2+2y^2-4x-8y+2=0$

$\Rightarrow x^2+y^2-2x-4y+1=0$ ....(ii)

Putting the value of x in eq. (ii),

$1+y^2-2-4y+1=0$

$\Rightarrow y^2-4y=0$

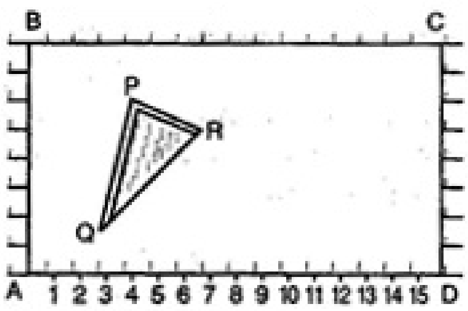
$\Rightarrow y(y-4)=0$

$\Rightarrow y$ = 0 or 4

19. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmoharare planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.

(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of $\triangle$PQR if C is the origin? Also calculate the area of the triangle in these cases. What do you observe?



Ans. (i) Taking A as the origin, AD and AB as the coordinate axes. Clearly, the points P, Q and R are (4, 6), (3, 2) and (6, 5) respectively.

(ii) Taking C as the origin, CB and CD as the coordinate axes. Clearly, the points P, Q and R are given by (12, 2), (13, 6) and (10, 3) respectively.

We know that the area of the triangle = $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\therefore$ Area of ΔPQR (First case) = $\frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$

= $\frac{1}{2}\left[4(-3)+3(-1)+6(4)\right]$

= $\frac{1}{2}\left[-12-3+24\right]$ = $\frac{9}{2}$ sq. units

And Area of ΔPQR (Second case) = $\frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$

= $\frac{1}{2}\left[12(3)+13(1)+10(-4)\right]$

= $\frac{1}{2}\left[36+13-40\right]$ = $\frac{9}{2}$ sq. units

Hence, the areas are same in both the cases.

20. ABCD is a rectangle formed by joining points A$(-1,-1)$, B$(-1,4)$, C$(5,4)$ and D $(5,-1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? Or a rhombus? Justify your answer.

Ans. Using distance formula, PQ = $\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$

= $\sqrt{9+\frac{25}{4}}$ = $\sqrt{\frac{61}{4}}$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$RS = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$SP = \sqrt{(-1-2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$\Rightarrow$ PQ = QR = RS = SP

Now, $PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = \sqrt{36} = 6$

And $SQ = \sqrt{(2-2)^2 + (4+1)^2} = \sqrt{25} = 5$

$\Rightarrow$ PR $\neq$ SQ

Since all the sides are equal but the diagonals are not equal.

$\therefore$ PQRS is a rhombus.

21. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?"Chameli disagrees. Using distance

formula, find which of them is correct.

Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[6-3]^2+[7-4]^2}$

$=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

BC = $\sqrt{[9-6]^2+[4-7]^2}$

$=\sqrt{(3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

CD = $\sqrt{[6-9]^2+[1-4]^2}$

$=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

DA = $\sqrt{[6-3]^2+[1-4]^2}$

$=\sqrt{(3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

Therefore, All the sides of ABCD are equal here. … (1)

Now, we will check the length of its diagonals.

AC = $\sqrt{[9-3]^2+[4-4]^2}$

$=\sqrt{(6)^2+(0)^2}=\sqrt{36+0}=6$

BD = $\sqrt{[6-6]^2+[1-7]^2}$

$=\sqrt{(0)^2+(-6)^2}=\sqrt{0+36}=\sqrt{36}=6$

So, Diagonals of ABCD are also equal. … (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

22. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (–3, 0)

(ii) (–3, 5), (3, 1), (0, 3) , (–1, –4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let A = (–1, –2), B = (1, 0), C= (–1, 2) and D = (–3, 0)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[1-(-1)]^2+[0-(-2)]^2}$

$=\sqrt{(2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

BC= $\sqrt{[-1-1]^2+[2-0]^2}$

$=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

CD= $\sqrt{[-3-(-1)]^2+[0-2]^2}$

$=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

DA= $\sqrt{[-3-(-1)]^2+[0-(-2)]^2}$

$=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

Therefore, all four sides of quadrilateral are equal. … (1)

Now, we will check the length of diagonals.

AC= $\sqrt{[-1-(-1)]^2+[2-(-2)]^2}$

$=\sqrt{(0)^2+(4)^2}=\sqrt{0+16}=\sqrt{16}=4$

BD= $\sqrt{[-3-1]^2+[0-0]^2}$

$=\sqrt{(-4)^2+(0)^2}=\sqrt{16+0}=\sqrt{16}=4$

Therefore, diagonals of quadrilateral ABCD are also equal. ... (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (–3, 5), B= (3, 1), C= (0, 3) and D= (–1, –4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB= $\sqrt{[3-(-3)]^2+[1-5]^2}=\sqrt{(6)^2+(-4)^2}=\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$

BC= $\sqrt{[0-3]^2+[3-1]^2}=\sqrt{(-3)^2+(2)^2}=\sqrt{9+4}=\sqrt{13}$

CD= $\sqrt{[-1-0]^2+[-4-3]^2}=\sqrt{(-1)^2+(-7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$

DA= $\sqrt{[-1-(-3)]^2+[-4-5]^2}=\sqrt{(2)^2+(-9)^2}=\sqrt{4+81}=\sqrt{85}$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD.

(iii) Let A = (4, 5), B= (7, 6), C= (4, 3) and D= (1, 2)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB= $\sqrt{[7-4]^2+[6-5]^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$

BC= $\sqrt{[4-7]^2+[3-6]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD= $\sqrt{[1-4]^2+[2-3]^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

DA= $\sqrt{[1-4]^2+[2-5]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Here opposite sides of quadrilateral ABCD are equal. … (1)

We can now find out the lengths of diagonals.

AC= $\sqrt{[4-4]^2+[3-5]^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

BD= $\sqrt{[1-7]^2+[2-6]^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here diagonals of ABCD are not equal. … (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

23. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of Δ ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

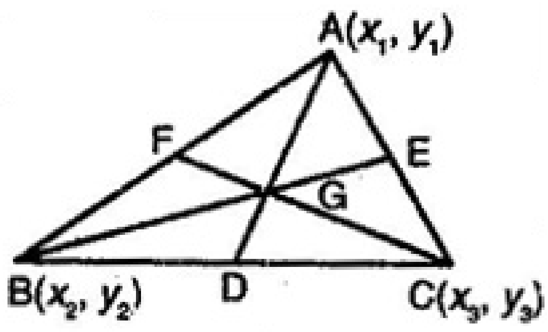
(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What do you observe?

(Note: The point which is common to all the three medians is called *centroid* and this point divides each median in the ratio 2 : 1)

(v) If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of Δ ABC, find the coordinates of the centroid of the triangle.



Ans. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of $\triangle$ABC.

(i) Since AD is the median of $\triangle$ABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since P divides AD in the ratio 2 : 1

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Since BE is the median of $\triangle$ABC.

$\therefore$ E is the mid-point of AD.

$\therefore$ Its coordinates are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Since Q divides BE in the ratio 2 : 1.

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Since CF is the median of $\triangle$ABC.

∴ F is the mid-point of AB.

∴ Its coordinates are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Since R divides CF in the ratio 2 : 1.

∴ Its coordinates are $\left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) We observe that the points P, Q and R coincide, i.e., the medians AD, BE and CF are concurrent at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.

(v) According to the question, D, E, and F are the mid-points of BC, CA and AB respectively.

∴ Coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Coordinates of a point dividing AD in the ratio 2 : 1 are

$$\left(\frac{1.x_1+2\left(\frac{x_2+x_2}{2}\right)}{1+2}, \frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$.

∴ The coordinates of a point dividing BE in the ratio 2 : 1 are

$$\left(\frac{1.x_2+2\left(\frac{x_1+x_3}{2}\right)}{1+2},\frac{1.y_2+2\left(\frac{y_1+y_3}{2}\right)}{1+2}\right)$$

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Similarly the coordinates of a point dividing CF in the ratio 2 : 1 are

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Thus, the point $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 2 : 1.

∴ The median of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$.